DO NOW

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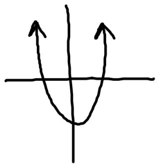
$g(x) = x^2 - 5$

domain: $(-\infty, \infty)$

range: $[-5, \infty)$



1.5 <u>Inverse Functions</u>

Formed by: interchanging the x and y coordinates of the ordered pairs

Ex: $f(x) = \{(1,4), (2, 5), (3, 6), (4, 7)\}$

inverse function:

$f^{-1}(x) = \{(4,1), (5,2), (6,3), (7,4)\}$

Definition of Inverse Function

A function g is the inverse of the function f if

$f(g(x)) = x$ and $g(f(x)) = x$

for each x in the domain of f and g.

Important observations about inverse functions:

1. If f is the inverse of g, then g is the inverse of f.
2. The domain of $f^{-1}(x)$ = the range of $f(x)$
   The range of $f^{-1}(x)$ = the domain of $f(x)$
3. A function need not have an inverse but if it does → it is unique.
4. The graph of $f^{-1}(x)$ is the reflection of the graph of $f(x)$ over the line $y=x$

Examples: Show that f and g are inverse functions.

pg 44; 2 $f(x) = 3 - 4x$ $g(x) = (3 - x)/4$

Prove: $f(g(x)) = x$ and $g(f(x)) = x$

$3 - \cancel{4}\left(\frac{3-x}{\cancel{4}}\right)$

$3 - (3 - x)$

$3 - 3 + x$

x

$\frac{3-(3-4x)}{4}$

$\frac{3-3+4x}{4}$

$\frac{4x}{4}$

x

Reflective Property of Inverse Functions

The graph of $f(x)$ contains the point $(a,b) \leftrightarrow$ the graph of $f^{-1}(x)$ contains the point (b,a)

Existence of an Inverse Function

A function has an inverse $\longleftrightarrow$ it is one-to-one.

One to One Function -

For every value of y, there is <u>EXACTLY</u> one corresponding value of x

* Must pass vertical line test
  AND
  horizontal line test.

Guidelines for finding the Inverse of a function

1. Determine whether the function has an inverse.

2. Solve for x as a function of y.

3. Interchange x & y
   $y = f^{-1}(x)$

4. The domain of $f^{-1}(x)$ is the range of $f(x)$

5. Verify $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$

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pg 44; 32 Find the inverse function of *f*.

$f(x) = x^3 - 1$

$f^{-1}(x) = \sqrt[3]{x+1}$

$y = x^3 - 1$

$x = y^3 - 1$ (switch x & y)

$x + 1 = y^3$ (solve for y)

$\sqrt[3]{x+1} = y$

Verify:

$f^{-1}(f(x)) = x$

$\sqrt[3]{x^3 - 1 + 1}$

$\sqrt[3]{x^3}$

x ✓

$f(f^{-1}(x)) = x$

$\left(\sqrt[3]{x+1}\right)^3 - 1$

$x + 1 - 1$

x ✓

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